HOMEWORK 4 – ANSWERS TO (MOST) PROBLEMS

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Section 2.7: Derivatives and rates of change

2.7.40. $f'(t) = -\frac{1}{t^2} - 1$ (show this, using the **definition** of the derivative) Velocity $= f'(5) = -\frac{26}{25}$ meters per second, Speed $= \frac{26}{25}$ meters/second

2.7.48.

(a) Rate of bacterias/hour after 5 houts
(b) $f'(10) > f'(5)$ (basically, the more bacteria there are, the more can be produced). But if there's a limited supply of food, we get that $f'(10) < f'(5)$, i.e. bacterias are dying out because of the limited supply

Section 2.8: The derivative as a function

2.8.3.

(a) II
(b) IV
(c) I
(d) III

2.8.23. $f'(t) = 5 - 18t$

2.8.40. -1 (not continuous there); 2 (graph has a kink)

2.8.45.

(a) Acceleration
(b) Velocity
(c) Position

2.8.54. Not differentiable at the integers, because not continuous there; $f'(x) = 0$ for x not an integer, undefined otherwise. Graph looks like the 0-function, except it has holes at the integers.

Section 3.1: Derivatives of polynomials and exponential functions

3.1.20. $S'(R) = 8\pi R$

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3.1.35. $y' = 4x^3 + 2e^x$, so $y'(0) = 2$, and so the equation of tangent line is $y - 2 = 2(x - 0)$, i.e. $\boxed{y = 2x + 2}$ and equation of normal line is $y - 2 = -\frac{1}{2}(x - 0)$, i.e. $\boxed{y = -\frac{1}{2}x + 2}$ (remember that the normal line still goes through $(0, 2)$, but has slope = the negative reciprocal of the slope of the tangent line)

3.1.47.

(a) $v(t) = s'(t) = 3t^2 - 3$; $a(t) = v'(t) = 6t$
(b) $a(2) = 12$
(c) $v(t) = 0$ if $t = 1$ or $t = -1$, but $t > 0$ (negative time doesn't make sense), so $t = 1$, and $a(1) = 6$

3.1.54. First of all $y' = \frac{3}{2}\sqrt{x}$ and first find a point x where $y'(x) = 3$ (remember that two lines are parallel when their slopes are equal, and the slope of $y = 1+3x$ is 3). So you want $\frac{3}{2}\sqrt{x} = 3$, so $\sqrt{x} = 2$, so $x = 4$. Now all that you need to find out is the slope of the tangent line to the curve at 4. The equation is: $\boxed{y - 8 = 3(x - 4)}$ (because from the above calculation the slope is 3, and the tangent line goes through $(4, f(4)) = (4, 8)$)

Section 3.2: The product and quotient rules

3.2.15. $y' = \frac{2t\left(t^4-3t^2+1\right)-\left(t^2+2\right)\left(4t^3-6t\right)}{\left(t^4-3t^2+1\right)^2}$

3.2.33. $y'(x) = 2e^x + 2xe^x$, so $y'(0) = 2$, and so the tangent line has equation: $y - 0 = 2(x - 0)$,i.e $\boxed{y = 2x}$, and the normal line has equation: $y - 0 = -\frac{1}{2}(x - 0)$, i.e. $\boxed{y = -\frac{1}{2}x}$

3.2.41. $f'(x) = \frac{2x(1+x)-x^2}{(1+x)^2} = \frac{x^2+2x}{x^2+2x+1}$, so $f''(x) = \frac{(2x+2)(x^2+2x+1)-(x^2+2x)(2x+2)}{(x^2+2x+1)^2}$, and so $f''(1) = \frac{(2+2)(1+2+1)-(1+2)(2+2)}{(1+2+1)^2} = \frac{(4)(4)-(3)(4)}{(4)(4)} = \frac{16-12}{16} = \frac{4}{16} = \boxed{= \frac{1}{4}}$

3.2.57. (9200)(30593) + (961400)(1400) = 1,627,000

Section 3.3: Derivatives of trigonometric functions

3.3.37. We have $\sin(\theta) = \frac{x}{10}$, so $x = 10\sin(\theta)$, so $x'(\theta) = 10\cos(\theta)$, and $x'\left(\frac{\pi}{3}\right) = 10\cos\left(\frac{\pi}{3}\right) = \frac{10}{2} = \boxed{5}$

3.3.39. 3 (multiply the fraction by $\frac{3}{3}$ and use the fact that $\lim_{x\to 0} \frac{\sin(3x)}{3x} = 1$)

3.3.40. $\frac{4}{6} = \frac{2}{3}$ (multiply the numerator by $\frac{4}{4}$ and the denominator by $\frac{6}{6}$ and use the facts that $\lim_{x\to 0} \frac{\sin(4x)}{4x} = 1$ and $\lim_{x\to 0} \frac{\sin(6x)}{6x} = 1$)